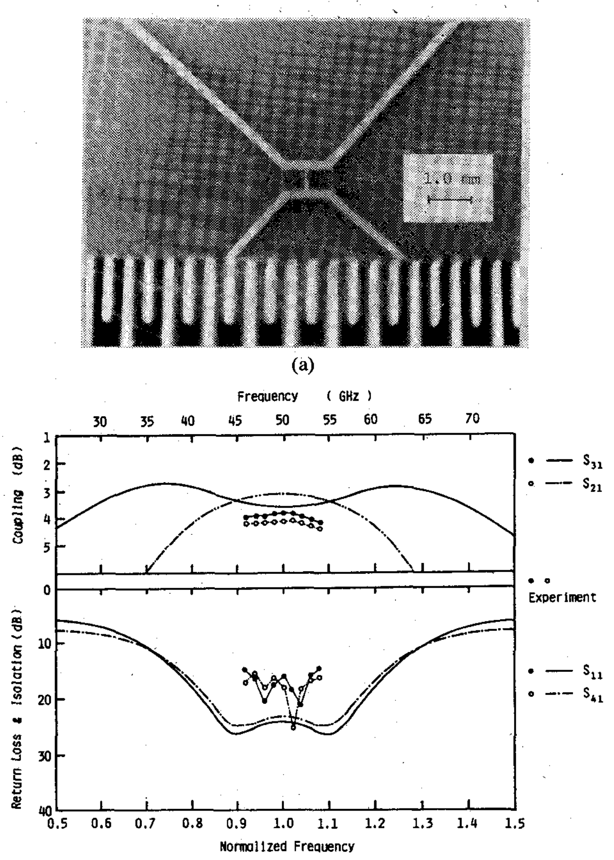

Fig. 8. (a) Photograph of the circuit pattern of the coupler 3-14 on a fine grained alumina ($\epsilon_r = 9.8$) substrate having a thickness of 0.2 mm. (b) Measured frequency characteristics of the coupler 3-14 in U-band (46–54 GHz).

due to conductor, not dielectric dissipation. Therefore, we assume that the electrical length has the following complex value:

$$\theta = \theta' - j\theta''$$

and

$$\theta''/\theta' = 0.005\sqrt{f/f_0}$$

as determined by experiments in C-band. The theoretical characteristics in Figs. 6 and 7 include the conductor loss. Therefore, the measured characteristics of the two devices in C-band showed good agreement with the theoretical ones. The frequency characteristics of the coupler 3-14 in U-band are shown in Fig. 8, together with a circuit pattern. As these characteristics include three waveguide-to-microstrip transitions and one matched load, the measured values in U-band were considerably worse than those in C-band, as shown in Fig. 6.

VI. Conclusions

Three-dB branch-line couplers with impedance ranges reduced to lie within the realizable range of microstrip line were presented. Couplers with more than four branches are not listed as they exhibit line impedances exceeding the upper limit of 160 Ω. The impedance ranges in the most practical cases of five- and six-branch couplers are 52 Ω–172 Ω and 51 Ω–208 Ω, respectively. Although only the 3-dB branch-line couplers were considered here, the design method itself is applicable to branch-line couplers with any degree and to various components in millimeter-wave IC.

Acknowledgment

The authors wish to express their thanks to the members of Fujitsu Laboratories, Ltd., for supplying several millimeter-wave components.

References

[1] H. Yatsuka, M. Ishizaki, T. Takano, and H. Komizo, "Millimeter-wave IC components using fine grained alumina substrate," in *1980 IEEE MTT Int. Microwave Symp. Dig.* (Washington, DC), May 1980, pp. 276–278.
[2] H. Komizo and Y. Tokumitsu, "Millimeter-wave integrated circuits," in *1981 IEEE MTT Int. Microwave Symp. Dig.* (Los Angeles, CA), June 1981, pp. 179–181.
[3] H. Ogawa, M. Akaike, M. Aikawa, T. Karaki, and J. Watanabe, "A 26-GHz band integrated circuit of a double-balanced mixer and circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 34–41, Jan. 1982.
[4] R. Levy and L. Lind, "Synthesis of symmetrical branch-guide directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 80–89, Feb. 1968.
[5] R. Levy, "Directional couplers," in *Advances in Microwaves.* New York: Academic, vol. I, 1966, pp. 115–209.
[6] A. Hislop and D. Rubin, "Suspended substrate *Ka*-band multiplexer," *Microwave J.*, pp. 73–77, June 1981.
[7] R. Levy, "Zolotarev branch-guide couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 95–99, Feb. 1973.
[8] R. Levy and J. Helszajn, "Specific equations for one and two section quarter-wave matching networks for stub-resistor loads," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 55–62, Jan. 1982.
[9] R. A. Pucel, "Design considerations for monolithic microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 513–534, June 1981.
[10] G. D. Vendelin, "Limitations on stripline Q," *Microwave J.*, pp. 63–69, May 1970.
[11] M. V. Schneider, "Microstrip lines for microwave integrated circuits," *The Bell Syst. Tech. J.*, pp. 1421–1445, May–June 1969.
[12] M. J. D. Powell, "A method for minimizing a sum of squares of nonlinear functions without calculating derivatives," *Comput. J.*, vol. 7, pp. 303–307, 1965.
[13] T. Okoshi, T. Imai, and K. Ito, "Computer-oriented synthesis of optimum circuit pattern of 3-dB hybrid ring by the planar circuit approach," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 194–202, Mar. 1981.

Characteristic Impedance of an Oval Located Symmetrically between the Ground Planes of Finite Width

K. V. SESHAGIRI RAO, B. N. DAS, AND A. K. MALLICK

Abstract **—A conformal transformation for the analysis of a transmission line with an oval-shaped center conductor symmetrically placed between two finite ground planes is developed. The formulation is used to calculate**

Manuscript received October 19, 1982; revised March 21, 1983.
K. V. Seshagiri Rao is with the Center for Research and Training in Radar and Communication, Indian Institute of Technology, Kharagpur-721302, India.
B. N. Das and A. K. Mallick are with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur-721302, India.

0018-9480/83/0800-0678$01.00 ©1983 IEEE

the characteristic impedances of oval, elliptic, circular, and planar conductors placed midway between the finite ground planes. The results on impedance are presented for various values of ground plane width to spacing ratios.

I. Introduction

Analysis of transmission lines having conductors of circular cross section located symmetrically between infinite ground planes has been carried out by Wheeler [1] and Crystal [2]. The analysis of a transmission line having a conductor of elliptic cross section asymmetrically located between infinite parallel plates has also been reported [3]. Furthermore, the authors recently developed a conformal transformation for the analysis of an oval symmetrically located inside a rectangular boundary [4].

In the present work, a generalized conformal transformation using the same mathematical method formulated by the authors [4] is developed for the analysis of a transmission line with an oval-shaped center conductor located symmetrically between two parallel planes of finite width. This generalized formulation is then used to determine the characteristic impedances of elliptic, circular, and planar conductors between these two finite ground planes.

The parametric equations that describe the oval-shaped boundary are obtained from the conformal transformation. The numerical data on the characteristic impedances for the above cases are presented.

II. General Analysis

Consider a center conductor that is in the form of an oval placed symmetrically between two finite parallel planes as shown in Fig. 1(a). From the results of the field distribution in the cross section of the stripline available in the literature, it is found that the field intensity on the ground plane decays very fast with an increase in distance from the center of the strip [8]. When the distance of a point from the center of the strip is 2.5 times the ground plane spacing, the field intensity decreases to 1 percent of its maximum value. Hence for ground planes having width equal to 2.5 times the ground plane spacing, the field intensity at the open end can be neglected for all practical purposes. It can, therefore, be assumed without appreciable loss of accuracy that for the scalar potential function Φ, $d\Phi/dn = 0$ along AB, BC, EF, and FG of Fig. 1(a). This is tantamount to the assumption that in the case of center conductor symmetrically located between the ground planes, the flux line symmetric with respect to the ground planes has a right angle at the corner. The Schwarz-Christoffel transformation that transforms the hatched portion of Fig. 1(a) into the upper half of the t-plane in Fig. 1(b) is, hence, given by [5]

$$Z = C_1\left[\lambda F(\sin^{-1} t|m) - \frac{j}{\sqrt{1-m\alpha^2}} \cdot F\left(\sin^{-1}\sqrt{\frac{1-m\alpha^2}{1-mt^2}}\,\middle|\, g\right)\right] + C_2 \quad (1)$$

where C_1, C_2, λ, m, and α are constants, and

$$g = \frac{(1-m)}{(1-m\alpha^2)}, \qquad 0 < m < 1$$

and F is an incomplete elliptic integral of the first kind of a given argument and modulus [6].

The expressions for the constants C_1, C_2, λ, eccentricities e_1

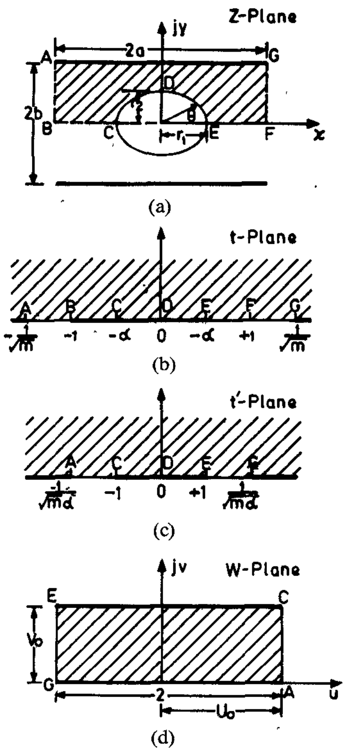

Fig. 1. Conformal representation of a transmission line with an oval-shaped center conductor between ground planes of finite width.

and e_2, r_1, and r_2 of Fig. 1(a), the b/a ratio as well as the parametric equations representing the curved boundary are of exactly the same form as those derived by the authors in the previous paper [4].

From numerical evaluations it is found that the parametric equations represent an ellipse for the values of b/a ranging from 0.2 to 0.5 and $0 < m\alpha^2 < 0.99$. For $0.99 < m\alpha^2 < 1$, the boundary of the center conductor assumes the form of an oval. A linear transformation of the form $t' = t/\alpha$ is used to transform the upper half of Fig. 1(b) to the upper half of Fig. 1(c) [7].

The transformation of the upper half of Fig. 1(c) into a parallel plate of Fig. 1(d) is given by [3], [4], [8]

$$W' = u + jv = C_3\int_0^{t'} \frac{dt'}{\sqrt{(1-t'^2)(1-m\alpha^2t'^2)}} + C_4$$
$$= C_3F(\Phi|m\alpha^2) + C_4 \quad (2)$$

where

$$t' = \sin\Phi.$$

From the substitution of the coordinates of the points A, C, E, G in t' and W' planes the transformation takes the form

$$W' = u + jv = \frac{-F\left(\sin^{-1}\frac{t}{\alpha}\middle|m\alpha^2\right)}{K(m\alpha^2)} + j\frac{K'(m\alpha^2)}{K(m\alpha^2)} \quad (3)$$

where $K(m\alpha^2)$ and $K'(m\alpha^2)$ are complete elliptic integrals of the first kind with the modulus $\sqrt{m}\,\alpha$ and $\sqrt{1-m\alpha^2}$, respectively, and

$$V_0 = \frac{K'(m\alpha^2)}{K(m\alpha^2)} \quad (4)$$

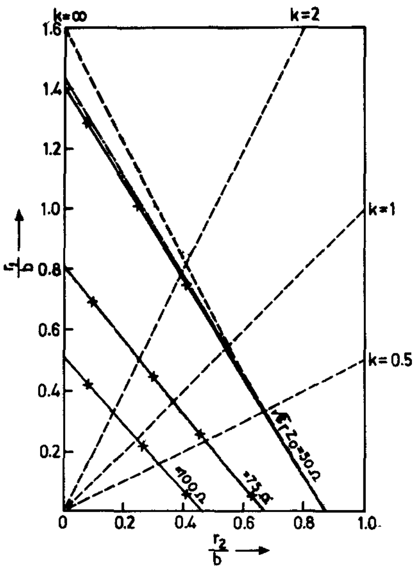

Fig. 2. Variation of the characteristic impedance of a transmission line as a function of conductor dimensions.(———— $a/b=2.0$; —·—·— $a/b=2.5$; × × × × $a/b=3.3$; and —— $a/b\geq 5.0$.)

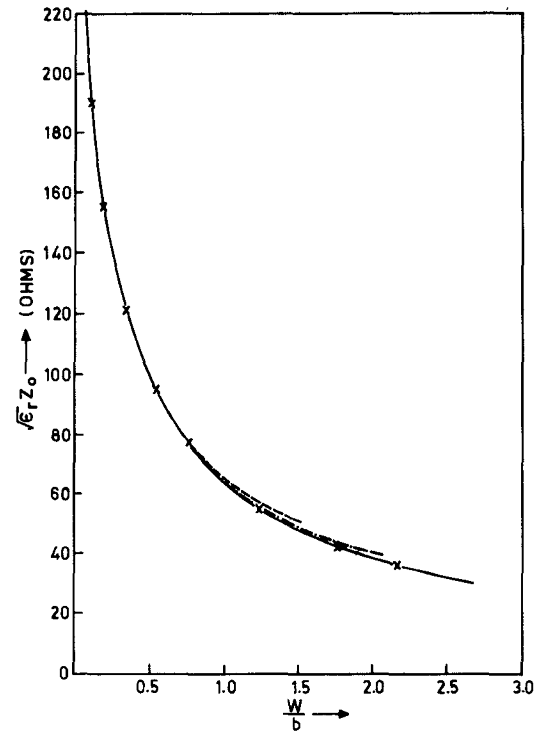

Fig. 3 Variation of the characteristics impedance of a stripline having finite ground planes as a function of w/b with a/b as parameter. (———— $a/b=2.0$; —·—·— $a/b=2.5$; × × × × $a/b=3.3$; and —— $a/b\geq 5.0$.)

III. Characteristic Impedance

The characteristic impedance of the line can be found from the formula

$$Z_0=\frac{30\pi}{\sqrt{\epsilon_r}}V_0=\frac{30\pi}{\sqrt{\epsilon_r}}\cdot\frac{K'(m\alpha^2)}{K(m\alpha^2)}. \tag{5}$$

For a given impedance of the line, the parameter $m\alpha^2$ can be found out from (5). Then for the known value of b/a and value of $m\alpha^2$ obtained above, the value of m can be determined from

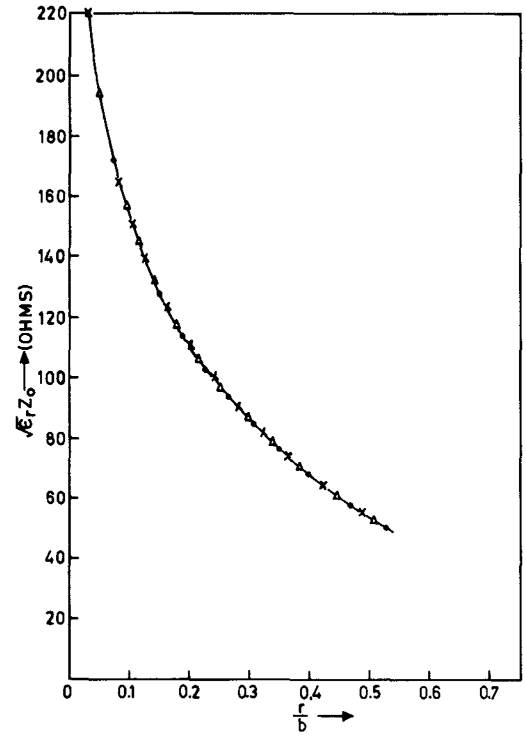

Fig. 4. Variation of the characteristics impedance of a transmission line with circular center conductor between finite ground planes as a function of the radius with a/b as parameter. (ΔΔΔΔ $a/b=2.0$; ···· $a/b=2.5$; × × × × $a/b=3.3$; and —— $a/b\geq 5.0$.)

the following equation:

$$\frac{b}{a}=\frac{K(g)F(\sin^{-1}\alpha|m)+\kappa\left[K(g)-F\left(\sin^{-1}\sqrt{1-m\alpha^2}|g\right)\right]K'(m)}{K'(g)F(\sin^{-1}\alpha|m)+\kappa\left[K(g)-F\left(\sin^{-1}\sqrt{1-m\alpha^2}|g\right)\right]K(m)} \tag{6}$$

where

$$\kappa(\text{compression ratio})=\frac{r_1}{r_2}\ (\text{Fig. 1(a)}).$$

After knowing the value of m, α can be found from the value of $m\alpha^2$ which is already known from (5). The data on the characteristic impedance are presented in Fig. 2 as constant impedance contours for various values of a/b (2 to 5). The variation of the characteristic impedance with the dimensions of the center conductor with a/b as parameters is presented in Figs. 3 and 4 for the cases of both planar and circular conductors respectively.

IV. Conclusions

The results presented in Fig. 2 reveal that as the ratio of ground plane width to ground plane spacing (a/b) is less than 2, some difference between the results obtained by the present method with those for infinite approximation $a/b\to\infty$ is observed. This difference is more pronounced for a planar strip conductor parallel to the ground planes having widths corresponding to impedances around 50 Ω and less. The above planar strip is the one of a degenerate ellipse with its major axis parallel to the ground planes. This deviation reduces as the eccentricity becomes zero. Further, it is also not noticeable for an elliptic conductor with its major axis perpendicular to the ground planes.

Acknowledgment

The authors would like to thank Prof. B. K. Sarap, Head of the Radar and Communication Center, for kindly extending the computational facility for this work.

References

[1] H. A. Wheeler, "The transmission-line properties of a round wire, between parallel planes," *IRE Trans. Antennas Propagat.*, vol. AP-3, pp. 203–207, Oct. 1955.
[2] E. G. Cristal, "Coupled circular cylindrical rods between parallel ground planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, July 1964.
[3] B. N. Das and K. V. Seshagiri Rao, "Analysis of an elliptical conducting rod between parallel ground planes by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1079–1085, July 1982.
[4] B. N. Das, K. V. Seshagiri Rao, and A. K. Mallick, "Analysis of an oval symmetrically located inside a rectangular boundary by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 403–406, May 1983.
[5] K. J. Binns and P. J. Lawrenson, *Analysis and Computation of Electric and Magnetic Field Problems*. New York: Pergamon Press, 1963.
[6] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Scientists*. New York: Springer, 1971.
[7] J. C. Tippet and D. C. Chang, "Radiation characteristics of electrically small devices in a TEM transmission cell," *IEEE Trans. Electromagn. Compat.*, vol. EMC-18, no. 4, pp. 134–140, Nov. 1976.
[8] J. S. Rao and B. N. Das, "Analysis of asymmetric stripline by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 299–303, Apr. 1979.

Spectral Domain Analysis of Interacting Microstrip Resonant Structures

ARVIND K. SHARMA, MEMBER, IEEE, AND BHARATHI BHAT, SENIOR MEMBER, IEEE

***Abstract*—An analysis of the interacting microstrip resonant structures, namely, the half-wave coupled and the quarter-wave coupled rectangular microstrip resonators is performed with the hybrid-mode formulation of the spectral domain technique. The resonant frequencies in the even and odd resonance modes are evaluated from the numerical solution of the characteristic equation. Results agree within ±1.5 percent of the experimental values.**

I. Introduction

Among various types of interacting resonant structures in microwave integrated circuit applications, the half-wave coupled and the quarter-wave coupled rectangular microstrip resonators are extensively used as network elements. In such structures, the propagation of waves is described in terms of the even and odd modes [1]. There is some difference between the even- and odd-mode phase velocities at lower microwave frequencies, but, as the frequency of operation increases, the divergence in the even- and odd-mode phase velocities becomes quite significant. In directional couplers, this causes degradation of match, directivity and isolation. It causes spurious response and reduces the operating bandwidth of filters. There have been many attempts, in the past, to investigate the effect of unequal phase velocities and to achieve their equalization [2]–[4].

In the above context, the study of interacting rectangular microstrip resonators assumes considerable importance. The early

Manuscript received December 29, 1982; revised April 12, 1983.
A. K. Sharma is with the Microwave Technology Center, RCA Laboratories, David Sarnoff Research Center, Princeton, NJ 08540.
B. Bhat is with the Center for Applied Research in Electronics, Department of Electrical Engineering, Indian Institute of Technology, New Delhi-110016, India.

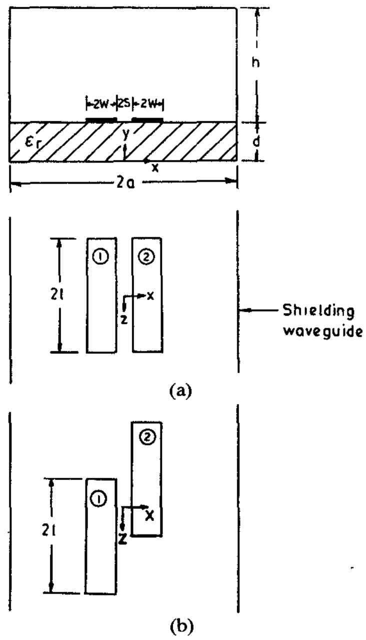

Fig. 1. Interacting rectangular microstrip resonators: (a) half-wave coupled parallel rectangular microstrip resonator; (b) quarter-wave coupled parallel rectangular microstrip resonator.

experimental work by Easter and Ritchings [5] provide some background information to assess the predictability of resonant frequencies in the even and odd resonance modes of a quarter-wave coupled rectangular microstrip resonator. An exact analysis of interacting resonant structures has been considered intrinsically difficult [6] and, therefore, there has not been any attempt to determine the resonant frequencies of the abovementioned structures. Consequently, various attempts to alleviate the effect of unequal phase velocities on the performance of the directional couplers and filters utilized the available information on the coupled microstrip lines.

In this paper, we have utilized the hybrid-mode formulation in the spectral domain to analyze these resonant structures. The divergence in the even- and odd-mode phase velocities of the abovementioned structures is thus determined in terms of their resonant frequencies.

II. Analysis

The interacting rectangular microstrip resonant structures in a shielding waveguide configuration are shown in Fig. 1. The basic building block in each case is a rectangular microstrip resonator of length $2l$ and width $2w$. The shielding waveguide has dimensions $2a$ and $d+h$. The dielectric substrate of relative permittivity ϵ_r has thickness d above the ground plane.

In the spectral domain analysis of the structure [7]–[11], the Fourier transform of the dyadic Green's functions are related to the transforms of the current densities on the conductors and the electric fields in the region of the interface complementary to the conductors, via the equation

$$\begin{bmatrix} \tilde{G}_{11}(\hat{k}_n,\beta,k_0) & \tilde{G}_{12}(\hat{k}_n,\beta,k_0) \\ \tilde{G}_{21}(\hat{k}_n,\beta,k_0) & \tilde{G}_{22}(\hat{k}_n,\beta,k_0) \end{bmatrix} \begin{bmatrix} \tilde{J}_{xc}(\hat{k}_n,\beta) \\ \tilde{J}_{zc}(\hat{k}_n,\beta) \end{bmatrix} = \begin{bmatrix} \tilde{E}_{zc}(\hat{k}_n,\beta) \\ \tilde{E}_{xc}(\hat{k}_n,\beta) \end{bmatrix} \quad (1)$$

0018-9480/83/0800-0681$01.00 ©1983 IEEE